

Math 315-003  
30–31 January 2004  
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Test  
Show relevant work!

Name \_\_\_\_\_

1. Define what it means for a subset  $A \subset \mathbf{R}$  to be bounded.
2. State the Completeness Axiom.
3. State the Archimedian Property.
4. Prove  $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$ .
5. Show a convergent sequence is bounded.
6. Show a bounded increasing sequence converges to its supremum.
7. Give the sequence definition of what it means for a function  $f : D \rightarrow \mathbf{R}$  to be continuous at a point  $x_0$ .
8. Give the epsilon-delta definition of what it means for a function  $f : D \rightarrow \mathbf{R}$  to be continuous at a point  $x_0$ .
9. If  $x_n \rightarrow A$  and  $y_n \rightarrow B$  are convergent sequences, show  $x_n y_n \rightarrow AB$ .
10. Show a continuous function on a closed interval is bounded above.
11. If  $0 < r < 1$ , show that  $\lim_{n \rightarrow \infty} r^n = 0$ .
12. Prove the Special Case of the Intermediate Value Theorem. If  $f : [a, b] \rightarrow \mathbf{R}$  is a continuous function with  $f(a) < 0$  and  $f(b) > 0$ , show there is a point  $x_0$  in the open interval  $(a, b)$  at which  $f(x_0) = 0$ .